Int. J. Solids Structures, 1967, Vol. 3, pp. 167 to 176. Pergamon Press Ltd. Printed in Great Britain

ON SECOND-ORDER STRAIN ACCUMULATION IN ALUMINUM IN REVERSED CYCLIC TORSION AT ELEVATED TEMPERATURES

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Abstract—Experiments on superpure aluminum in reversed cyclic torsion at elevated temperatures showed that the second-order axial elongation of the specimen is increasing with increasing temperature in the 25°–400°C temperature range.

The quadratic relation between the accumulated second-order strain increments and the applied shear strain amplitude already found at room temperature holds at each elevated temperature investigated. The accumulated second-order strain increments are functions of the applied shear strain amplitude, previous number of cycles and temperature. The cycle and temperature effects can be separated, both functions being exponential.

A fictitious activation energy was calculated for the second-order extension of aluminum and was found to be almost one order of magnitude smaller than the activation energy of static (high temperature) creep, showing that different material parameters are involved in the two phenomena.

1. INTRODUCTION

WHILE reversible second-order effects in isotropic elastic solids (Poynting effect) and in viscous and visco-elastic fluids (Weissenberg effect) are well known, it has been only recently discovered that irrecoverable second-order strain increments accumulate in the course of cyclic torsion of quasi-isotropic strain hardening metals [1–3]. The continuum mechanical theory of accumulating, irreversible second-order effects is given in [2]. Neither reversible nor irreversible second-order effects have been studied at temperatures different from room temperature. As a first approach an experimental study of the effect of temperature on irreversible second-order strain accumulation has been undertaken.

2. EXPERIMENTAL CONDITIONS

The experimental set-up is similar to that used in the investigation of the second-order effects at room temperature [3]. Cylindrical specimens of $\frac{7}{32}$ in. dia. and $L_0 = 1{\cdot}25$ in. gage length of 99·99% aluminum used for the experiments at elevated temperatures were annealed *in vacuo* for 5 hr at 350°C in order to obtain 0·1–0·2 mm grain size. The specimens were electropolished in a solution of five parts methanol to one part perchloric acid.

In the alternating torsion fatigue machine used for the tests [4], the total strain amplitude (plastic and elastic) could be varied by means of a variable eccentric. This torque was applied to one end of a specimen. The other end could not rotate but was free to slide in the longitudinal direction. The axial strain accumulating during the alternating torsion was measured by means of a micrometer attached to the frame of the machine and making contact with a screw attached to the sliding grips. Extensions could thus be determined as a function of the number of cycles. The testing machine had long stainless steel grips that held the specimen in the uniform-temperature zone of a hinged tubular furnace.

In view of the fact that even in an elastic medium a small longitudinal force can prevent second-order extension produced by an applied torque [5] the small friction forces in the sliding support cannot be neglected. An attempt was therefore made to find approximately the magnitude of the axial compressive force that would counteract second order axial elongation of a plastically deforming specimen. Tests were performed at 300°C and various axial loads were applied in addition to cyclic torsion of $\pm 2°$ (30×10^{-4} surface shear). The axial elongation was plotted as a function of the axial stress and the curve extrapolated to zero axial extension. As shown in Fig. 1, the compressive stress in the specimen that could annihilate the axial elongation is less than 100 psi, a magnitude that could be produced by friction in the machine.

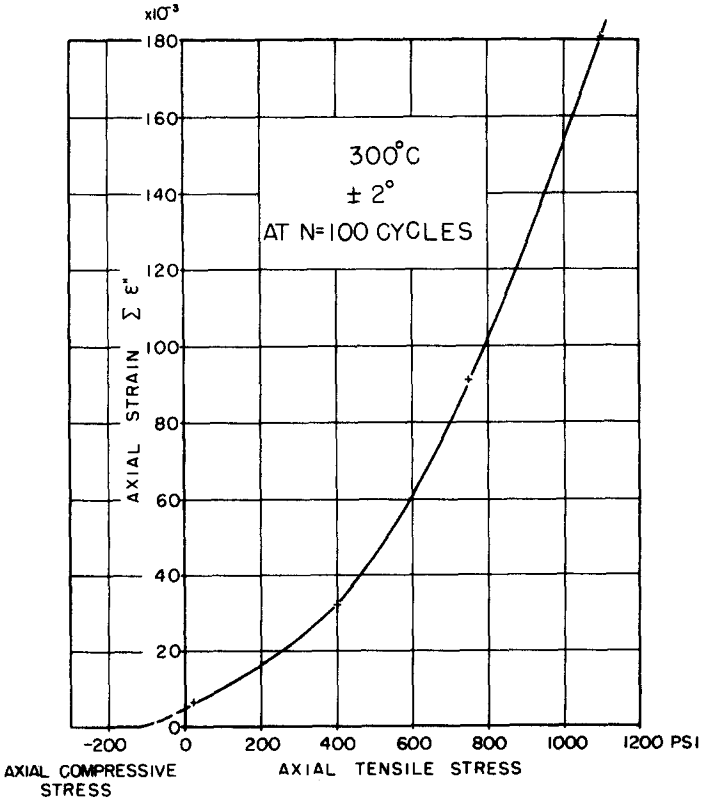

FIG. 1. Second-order axial strain accumulation $\Sigma\varepsilon''$ at $N = 100$ cycles as a function of the applied axial tensile stress for $\varphi = \pm 2°$ at 300°C. The extrapolated value at zero axial extension shows the compressive stress that could annihilate the axial elongation.

At cyclic torsion of $\pm 0{\cdot}5°$ roughly one-sixteenth of 100 psi compressive stress might prevent the axial extension since the second-order axial forces are in proportion to the squares of the applied shear. Therefore friction must be avoided in investigating second-order elongations; it was found that in the torsion fatigue machine used an axial tensile force equivalent to 25 psi axial tensile stress is needed to overcome the friction in the sliding parts of the machine. Hence all experiments in alternating torsion reported in the following were carried out by applying a force which, at zero friction, would produce 25 psi tensile stress in the axial direction, but actually ensures zero (compressive) stress in the specimen.

The tests were carried out at a frequency of 1750 c/min.

Test temperatures were 25°, 100°, 200°, 300°, and 400°C and at each temperature cyclic torsion-angles $\varphi = \pm 0{\cdot}5°$, $\pm 1°$, $\pm 2°$, $\pm 3°$ and $\pm 3{\cdot}5°$ were applied. With the above given dimensions of the specimens a torsion angle of $\varphi = \pm 1°$ produces $\gamma = \pm 15 \times 10^{-4}$ reversed shear strain.

In evaluating the data the logarithmic measure of strain $\varepsilon'' = \ln(1+E'')$ where $E'' = \Delta L/L_0$, ΔL denotes the plastic extension per cycle and ε'' denotes the plastic strain increment per cycle has been used, because in this strain-measure the addition of strain increment produces the correct measure of resulting strain.

3. AN EXAMPLE OF SECOND-ORDER STRAIN ACCUMULATION AT ELEVATED TEMPERATURES

The effect of temperature on the axial strain accumulation is shown in Fig. 2 as an example for $\varphi = \pm 2°$. The presented diagrams extend to fatigue fracture of the specimen. Arrows on the diagrams show the points where cracking or necking of the specimens began. It can be seen that the second-order strain accumulation increases with increasing temperature in the range of 25°C–400°C.

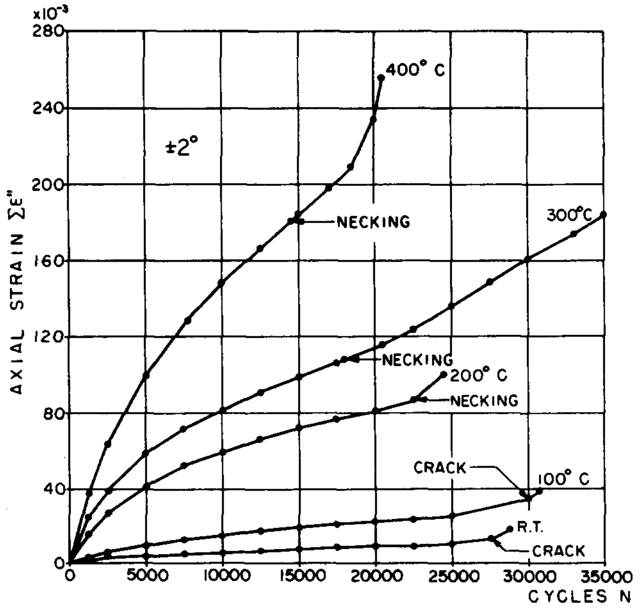

FIG. 2. Second-order axial strain accumulation $\Sigma\varepsilon''$ at various temperatures as a function of the number of reversed torsion cycles up to fatigue fracture of the specimen at $\varphi = \pm 2°$.

The rate of axial strain accumulation is largest at the beginning and decreases in the course of cycling at each temperature. While at small torsion angles and at low temperatures the permanent elongation of the specimen approaches a finite limit before fracture, it does not do so at higher temperatures at which the axial extension grows steadily with the number of torsion cycles applied. The larger the applied torsion-angle and the larger the temperature, the less the rate of axial strain accumulation decreases in the course of cycling.

Except for low temperatures and small torsion-angles the specimens contract locally before fracture; the number of cycles at which necking occurs is marked on the diagrams. Some of the specimens show double necking. Figure 3(a) is a photograph of a specimen cycled $\varphi = \pm 3{\cdot}5°$ torsion-angle at 300°C, the specimen showing double necking; the neck at which fracture occurred is down to a point. Figure 3(b) represents a specimen cycled with $\varphi = \pm 3{\cdot}5°$ at 100°C; this specimen fractured along the principal shear planes similarly to fracture at room temperature [3].

4. RELATION BETWEEN FIRST-ORDER TORSION STRAIN AMPLITUDE AND SECOND-ORDER AXIAL STRAIN INCREMENT AT ELEVATED TEMPERATURES

It has been shown in the previous investigation of second-order axial strain accumulation under cyclic torsion at room temperature that a quadratic relation between the second-order strain increment and the applied shear strain amplitude is characteristic of this phenomenon [2]. This is the same form of relation observed by Poynting [6] on elastic steel wires and interpreted by Reiner [7] as recoverable second-order axial extension. Distinguishing between geometric and tensorial non-linearity and between recoverable and irrecoverable strain, the following approximate equations can be assumed to describe the relation between first- and second-order strain:

the irrecoverable strain increment $\frac{1}{2}\varepsilon''$ per half cycle*

$$\tfrac{1}{2}\varepsilon'' = \text{const.}\,(\gamma^2-\gamma_e^2) = \tfrac{1}{12}(1+\varkappa)(\gamma^2-\gamma_e^2), \tag{1}$$

the recoverable strain $\frac{1}{2}\varepsilon'$ per half cycle

$$\tfrac{1}{2}\varepsilon' = \tfrac{1}{12}(1+\varkappa)\gamma_e^2 \tag{2}$$

where γ denotes the total γ_e the elastic shear strain amplitude and $\varkappa > 0$ is a coefficient which reflects the value of the parameter of the tensorial non-linearity (quadratic term) in the constitutive equation. For the elastic range in hard metals this factor has been found to be of the order of $4 < \varkappa < 8$ [6, 7], in the plastic range it is considerably smaller and decreases in importance with increasing shear strain.

Since in superpure aluminum the elastic part of the total strain is small,† the irreversible second-order axial strain increment per cycle can in first approximation be related to the applied shear strain by the quadratic function

$$\varepsilon'' = C\gamma^2 \tag{3}$$

where $C = C(N)$ is a decreasing function of the number N of previously applied cycles.

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* The strain increments accumulate at double the frequency of the applied strain cycle [2].

† See Fig. 15 in [8].

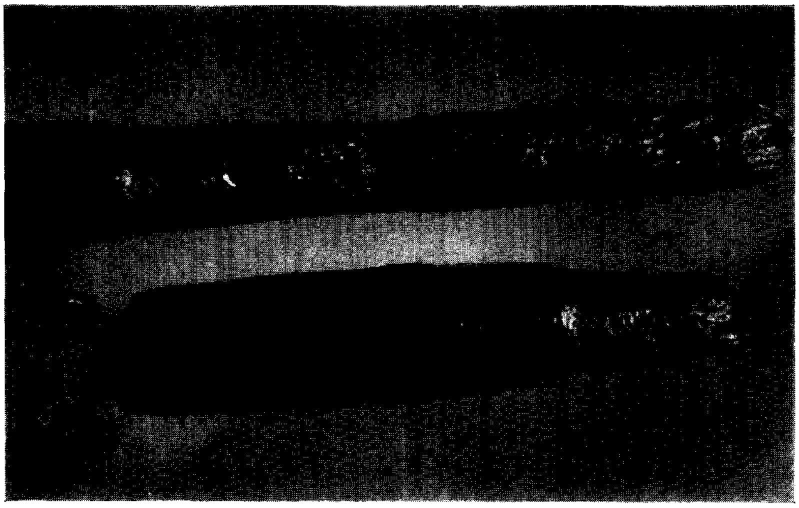

FIG. 3(a). Specimen cycled until fracture at $\varphi = \pm 3{\cdot}5°$ torsion angle at 300°C; the specimen shows double necking

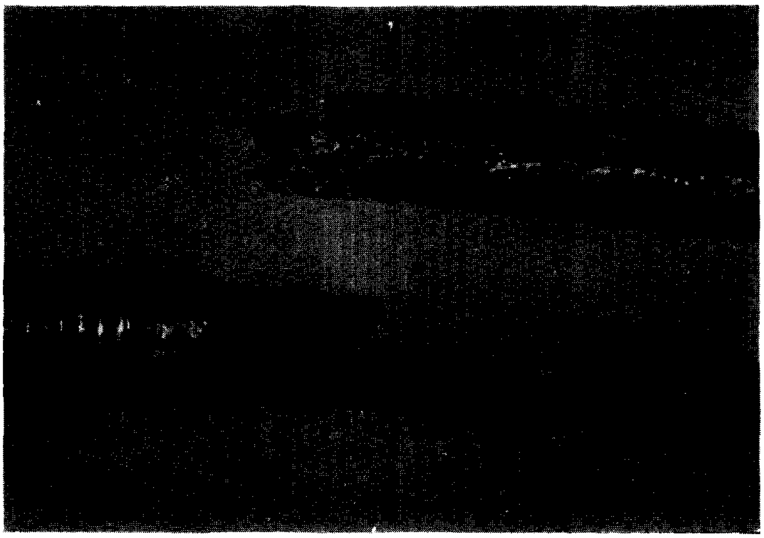

FIG. 3(b). Specimen cycled until fracture at $\varphi = \pm 3{\cdot}5°$ torsion angle at 100°C; the specimen fractured along the principal shear planes.

This formulation accounts for the fact that ε'' depends on the number of shear strain cycles that precede the one investigated since the second-order strain increments decrease with the number of cycles N, as seen in the diagrams of Fig. 2.

The accumulated second-order strain increments for N cycles can therefore be written

$$\Sigma\varepsilon''(N) = N\varepsilon''(N) = nC(N)\gamma^2 \tag{4}$$

This quadratic relation was found to hold for small strains at room temperature. Deviations from the quadratic relation for large shear strain occur because of the gradually decreasing rate of first-order strain-hardening as a function of the applied shear strain [2].

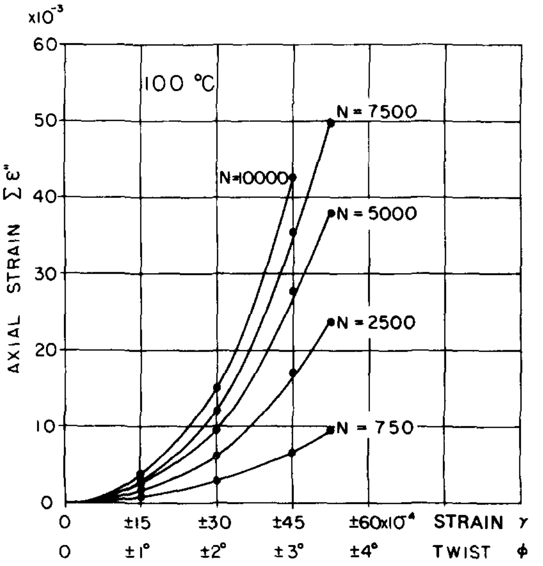

Fig. 4. Second-order axial strain accumulation $\Sigma\varepsilon''(N)$ at $N = 750, 2500, 5000, 7500$ and 10,000 cycles of reversed torsion as a function of the torsion angle at 100°C temperature.

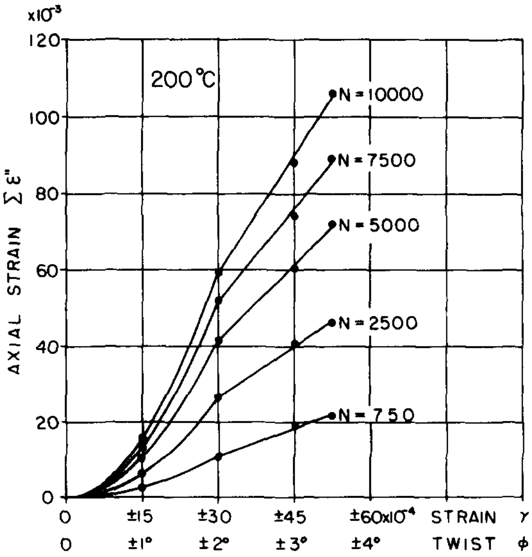

Fig. 5. Second-order axial strain accumulation $\Sigma\varepsilon''(N)$ at $N = 750, 2500, 5000, 7500$ and 10,000 cycles of reversed torsion as a function of the torsion angle at 200°C temperature.

Figures 4 and 5 show the relation between $\Sigma\varepsilon''$ and the applied total shear strain for N = 750, 2500, 5000, 7500 and 10,000 cycles for two of the temperatures applied. It can be seen from the diagrams that the quadratic relation for small strain amplitudes (until about ±2°) holds not only at room temperature but at each elevated temperature investigated. Beyond the range of the quadratic relation (above ±2°) the relation between axial strain and torsion angle is of higher than second power up to 100°C. At 200°C and above, however, the relation is nearly linear. Thus at small shear strains the quadratic relation seems to be valid for elevated temperatures.

An attempt has been made to separate the cycle and temperature dependence of the accumulated second-order axial strain. Figure 6 shows for various temperatures one-fourth of the axial strain as function of the number of torsion cycles in logarithmic representation for $\varphi = \pm 2°$. Data are given until the start of crack propagation or necking of the specimens. During the most important part of the life the function can be approximated by a straight line.

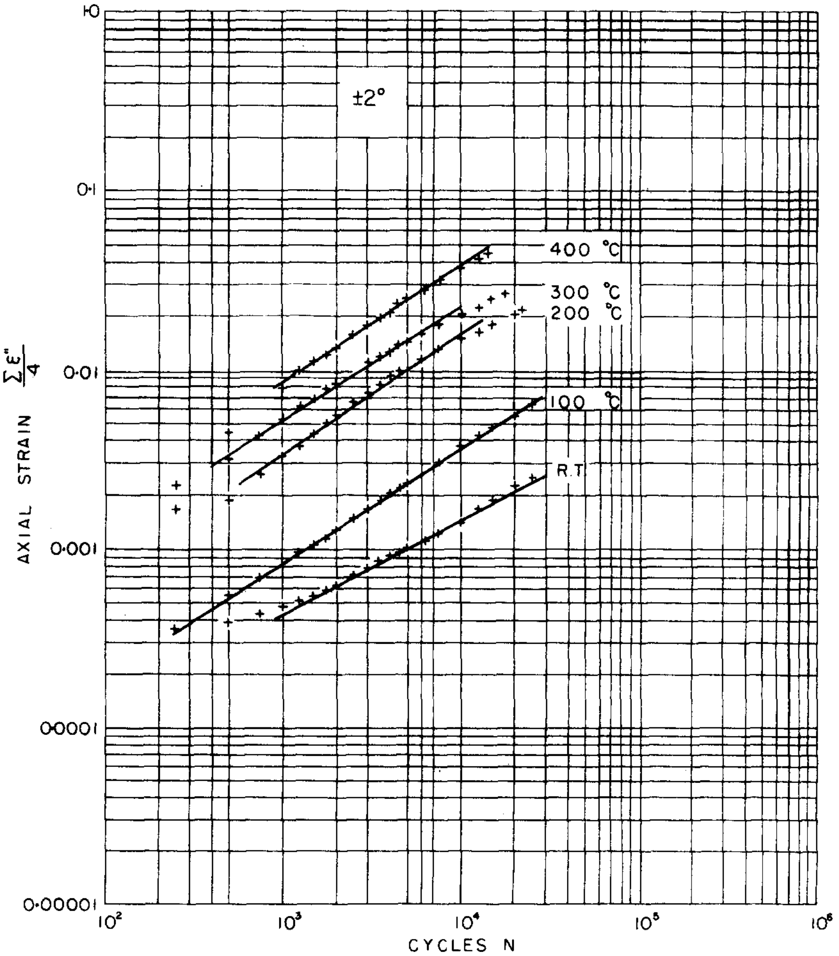

FIG. 6. One-fourth of second-order axial strain ($\Sigma\varepsilon''/4$) for $\varphi = \pm 2°$ at various temperatures as a function of the number of torsion cycles.

On the basis of Fig. 6, the cycle dependence of the accumulated second-order strain can be roughly approximated by

$$\Sigma\varepsilon'' = C_1 N^{\beta} \tag{5}$$

Values of C_1 and β for various temperatures are given in Table 1 for $\varphi = \pm 2°$. As β does not seem to change with temperature beyond experimental scatter the cycle dependence of second-order strain accumulation seems to be independent of temperature. The plotting of the logarithm of C_1 of equation (5) as a function of the logarithm of the absolute temperature gives a straight line which can be written in the form

TABLE 1. VALUES OF C_1 AND β AT VARIOUS TEMPERATURES FROM DATA FOR $\varphi = \pm 2°$

Temp. (°C)	Temp. (°K)	Slope β	Const. C_1
R.T.	293	0·56	0·0000124
100	373	0·662	0·00001
200	473	0·650	0·0000325
300	573	0·650	0·000070
400	673	0·638	0·000114

$$C_1 = C_2 T^{\alpha} \tag{6}$$

with values of $\alpha = 3{\cdot}80$ and $C_2 = 4{\cdot}4 \times 10^{-15}$. Thus the accumulated second-order axial strain can be roughly approximated by the equation

$$\sum^{N} \varepsilon'' = C_2 T^{\alpha} N^{\beta} \varphi^2 = C_3 T^{\alpha} N^{\beta} \gamma^2 \tag{7}$$

in the range of validity of the quadratic relation and for the frequency investigated.

5. FICTITIOUS ACTIVATION ENERGY FOR SECOND-ORDER EXTENSION

If second-order effects were controlled by thermal activation processes, the temperature dependence of the second-order strain could be expressed in the form

$$\varepsilon'' = \dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{\Delta H}{RT}\right) = C_4(N)\gamma^2 \exp\left(-\frac{\Delta H}{RT}\right) \tag{8}$$

On the basis of equation (8) calculations were made of a fictitious activation energy of second-order extension for no other reason than to show that the process of second-order extension is governed by a different temperature function than the first-order extension observed in "high-temperature" creep, and that different material parameters are involved.

The method based on equation (8) is the usual one for determining activation energy ΔH from creep tests by comparing strain rates in the range of steady state creep at $\varepsilon = \text{const.}$ for different temperatures.

By applying the method for the cycle dependent second-order strain accumulation, the comparisons of $\dot{\varepsilon}_1, \dot{\varepsilon}_2, \ldots$ at $T_1, T_2, \ldots$, must be made at N = const. because of the cycle dependence of C_4 in equation (8); thus, ε = const. can not be satisfied at the same time. Since the values of ε at which $\dot{\varepsilon}$ as a function of T has been considered are small (less than 2 percent) the method seems to be a satisfactory approximation.

For a torsion angle $\varphi = \pm 1°$ the average second-order strain rate was determined between 1000 and 2000 cycles at various temperatures and plotted against $1/T$(°K) in Fig. 7. The points lie with little scatter on a straight line. The average activation energy between room temperature and 400°C as taken from the slope of the line in Fig. 7 is $\Delta H \simeq 4500$ cal/mole. Although steady state considerations are not strictly applicable to cyclic strain accumulation, diagrams of second-order strain accumulation for small strain amplitudes show a kind of "steady state" strain accumulation before cracking occurs.

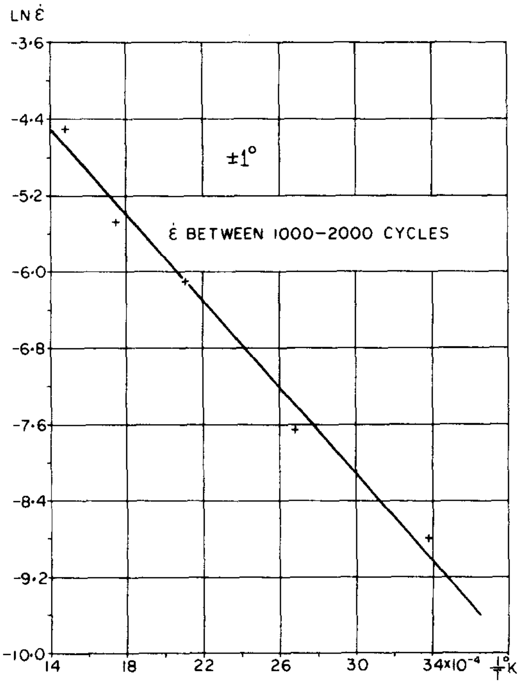

FIG. 7. Average second-order strain rate between 1000 and 2000 cycles at $\varphi = \pm 1°$ as a function of absolute temperature.

The fictitious activation energy of second-order extension was also calculated from strain rates in the "steady state" range for torsion angle $\varphi = \pm 1°$. Calculations based on observations in the 40,000–50,000 cycle range gave an activation energy similar to that calculated from strain rates between 1000 and 2000 cycles.

Activation energy for "high-temperature" creep for high purity aluminum has been determined by Dorn *et al.* [9] as a function of temperature. It has been found that the highest value of about 35,500 cal/mole spans a range of temperatures from about 500°K to 850°K. This coincides with the estimated activation energy for self-diffusion in aluminum [10]. The next lower plateau, spanning the range from about 250°K to about

375°K, has an activation energy of about 27,500 cal/mole. Between 375°K and 500°K the apparent activation energy increases gradually from 27,500 to 35,000 cal/mole.

As there is almost an order of magnitude difference between these activation energies of static "high-temperature" creep and the fictitious activation energy of cyclic second-order extension, it appears that different material parameters are involved in the two phenomena, and that the process of "cyclic creep" is much less temperature sensitive and depends on physical constants that are different from those of static creep.

6. CONCLUSIONS

The reported investigation of the second-order axial strain accumulation in superpure aluminum subject to cyclic torsion shows that the second-order strain increases significantly with increasing temperature in the range of 25°–400°C. This temperature dependence is, however, governed by physical parameters that are different from those governing first order rate processes in the metal.

In the interpretation of the experimental results the cycle and temperature effects on the second-order strain accumulation have been expressed for a single frequency. This is by no means adequate, since any quasi-viscous component of the shear deformation is likely to be associated with a viscous component of the second-order strain increment which would necessarily be frequency dependent. Such discrimination between the plastic (frequency independent) and the quasi-viscous (frequency dependent) component of second-order strain increment requires, however, both a broadening of the theory of second-order strain accumulation to include the viscous in addition to the strain-hardening response in the underlying constitutive equation of the medium, as well as a new extensive series of experiments at sufficiently different frequencies of cyclic torsion over a sufficiently wide range of temperatures. The results of such an investigation, which is being carried out at present, will be reported in a subsequent paper.

Acknowledgement—The assistance of Mr. J. Liu in the performance of the experiments is gratefully acknowledged.

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(*Received* 4 *February* 1966; *revised* 11 *May* 1966)

Résumé—Des expériences sur de l'aluminium superpure en état de torsion cyclique inverse, faites à températures élevées ont démontré que l'élongation axiale de second ordre du specimen augmente à mesure que la température s'élève entre 25 et 400°C. La relation de second degré entre les accroissements de tension accumulés de second ordre et l'amplitude de l'effort de cisaillement appliqué dejà trouvée à la température ambiante se maintient à chaque élevation de température étudiée. Les accroissements de tension accumulés de second ordre sont fonctions de l'amplitude de l'effort de cisaillement appliqué, du nombre de cycles et de la température. Les effets du cycle et de la température peuvent être séparés, ces deux fonctions étant exponentielles.

Une énergie activante factice a été calculée pour l'allongement de second ordre de l'aluminium, et l'on a trouvé qu'elle était presque d'un ordre de grandeur plus petite que l'énergie activante d'un cheminement statique (température élevée), indiquant ainsi que différents paramètres de matière sont mêlés aux deux phénomènes.

Zusammenfassung—Versuche an hochreinem Aluminium bei erhöhter Temperatur mit umgekehrter zyklischer Torsion zeigten, daß Axialverlängerung zweiter Ordnung des Prüflings im Bereich von 25° bis 400° mit ansteigender Temperatur zunimmt.

Das quadratische Verhältniß zwischen den angesammelten Spannungszunahmen zweiter Ordnung und der angewandten Scherbeanspruchung die bei Raumtemperatur festgestellt wurde, zeigte sich auch bei allen anderen Temperaturen, die untersucht wurden. Die angesammelten Spannungszunahmen sind Funktionen der angewandten Scherbeanspruchungsamplituden, wie auch der vorhergehenden Zyklen und der Temperatur. Die Einflüße der Zyklen sowie der Temperatur können abgesondert werden, beide sind exponential.

Die fiktive Aktivationsenergie für die Streckung zweiter Ordnung für Aluminium wurde errechnet, diese war um eine Größenordnung kleiner als die statische Kriechenergie (bei hoher Temperatur), das zeigt, daß verschiedene Eigenschaften diese Erscheinungen bewirken.

Абстракт—Опыты с чистым беспримесным алюминием в обратном циклическом кручении при повышенных температурах показали, что осевое удлинение второго порядка образца увеличивается с увеличением температуры в температурном диапазоне 25°–400°С.

Квадратное отношение между накопленными приращениями напряжения второго порядка и применённой амплитудой напряжения сдвига, уже найденное при комнатной температуре, удерживается при каждом исследуемом повышении температуры. Накопленные приращения деформации второго порядка представляют функции применённой амплитуды напряжения сдвига, предыдущих количеств циклов и температуры. Эффекты цикла и температуры могут быть разделены, обе функции—показательны.

Энергия фиктивного активирования была расчитана для расширения второго порядка алюминия и было найдено, что она почти на один порядок величины меньше, чем энергия активирования статической ползучести (высокая температура), что указывает на то, что в двух явлениях вовлечены различные параметры материала.